Languages

Exam 1, Second semester 2014/2015, Form: A

Section: 01

32/35

## Section 1. (1 point each)

Mark the following statements with **True** if they are true and **False** otherwise.

10

T The pumping lemma can be used to prove that the following language is non-regular $L = \{a^p :$ $p$ is a prime number$\}$.

T The language $L = \{w \in \{a, b\}^+ : |w| \mod 971 = 0\}$ can be generated using a regular expression.

F Nondeterminisim is useless since every NFA can be represented by an equivalent DFA.

T The language $L = \{w \in \{a, b\}^* : w$ has at least 3 $a$'s and every $a$ is followed by at most two $b$'s$\}$ is regular.

T Given two regular languages $L_1$ and $L_2$, the language $L_1 - (L_2 \cup L_1)^*$ is also regular.

T The grammar $S \to aS|aaS|aaaS|A; A \to \lambda|bbA$ is a regular grammar.

T Every regular language can be generated by a right-linear grammar.

F The language $L = \{a^n b^m c^{n+m} : n, m \geq 0\}$ is regular.

F The following grammar $S \to aS|Sbbb|b$ represents the language $L = \{a^n b^{3n+1} : n \geq 0\}$.

F The regular expression $(00)^*(11)^*1$ generates the language that contains all strings with an even number of 0's and an odd number of 1's. → ( 1 1 1 0 0 ) is string with even number of 0's and odd number 1's but we can't generates by this langeage, This language give as: all string with an even number of 0'S Follow by odd number of 1's (the zeros must come [illegible]

## Section 2. (5 points each)

1. Consider the following language

$$L = \{w \in \{a, b\}^* : w \text{ starts and ends with different letters}\}.$$

Show that $L$ is a regular language.

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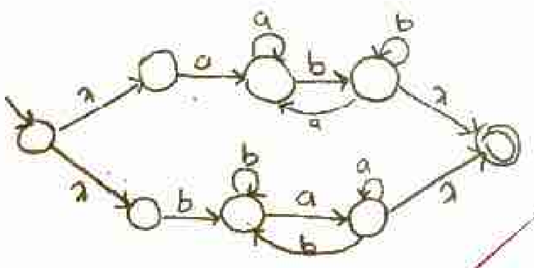



∵ we can Draw NFA

∴ L is regular

Ex:

abbbab ✓

baaaba ✓

2. Assume that the symbols in a word $w$ are numbered as $w = x_1 x_2 x_3 \ldots$ (i.e. $x_1$ is at position 1, etc.). Find a regular expression for the language

$$L = \{w \in \{0,1\}^* : \text{every odd position in the word is } 1\}.$$

$r = 1 + (10)^*$ $\xrightarrow{Ex}$ 1

101

1010

10101

2

3. If the following language is regular show that it is regular, otherwise prove that it is not regular.

$$L = \{www : w \in \{a,b\}^*\}.$$

5

Let $w = a^m b^m a^m b^m a^m b^m$

$= xyz$

since $|xy| \leq m$

and $|y| \geq 1$

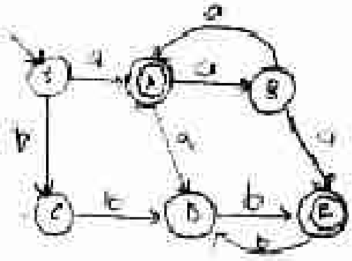
$y = a^k \qquad 1 \leq k \leq m$

$w_2 = a^{m+k} b^m a^m b^m a^m b^m \notin L \quad , k \geq 1$

$\therefore$ By the pumping lemma $L$ is not regular

4. Construct a regular grammar for the following language

$$L = \{a^n b^m : n + m \text{ is odd}\}$$



$S \rightarrow aA \mid bC$

$A \rightarrow aB \mid aD \mid \lambda$

$B \rightarrow aA \mid aE$

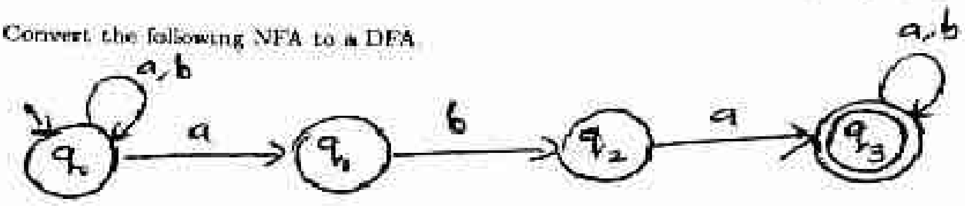
$C \rightarrow bD$

$D \rightarrow bE$

$E \rightarrow bD \mid \lambda$

5

5. Convert the following NFA to a DFA

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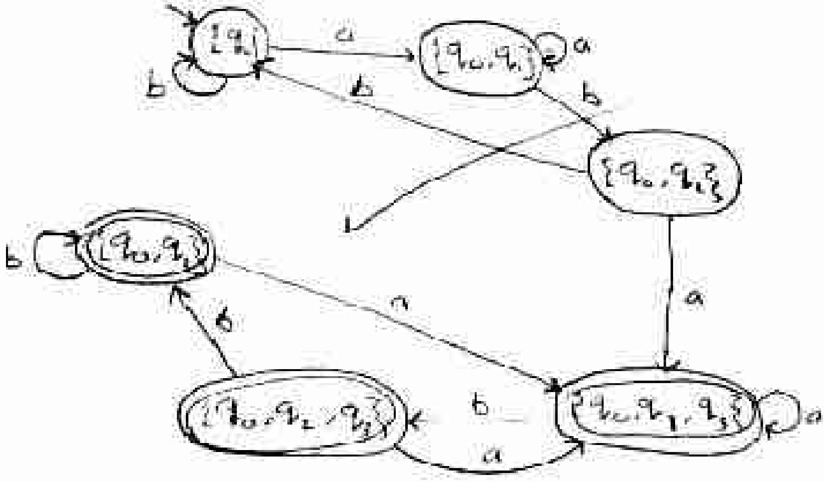


CSC 314: Formal Languages
Exam 2, Second semester 2014/2015, Form: $\boxed{A}$

Student Number:
Section: 01

29.5 / 35

Section 1. (1 point each)

Mark the following statements with **True** if they are true and **False** otherwise.

6

T A grammar $G = (V, T, S, P)$ is context-free if all productions have the form $A \rightarrow x$ where $A \in V$ and $x \in T^*$.

F A grammar is ambiguous if you can derive two different words from its productions using a derivation tree or a left-most derivation or a right-most derivation.

T Brute force can be used for parsing and answering the membership question if we eliminate unit and $\lambda$ productions from the grammar.

T Every NPDA has a stack, and words are only accepted when the NPDA moves into a final state and the stack is empty.

T The grammar $S \rightarrow aSb|ab$ is equivlant to $S \rightarrow aAb, A \rightarrow aAb|\lambda$.

T The intersection of two context free grammars is always context free.

T The language $L = \{ww : w \in \{0,1\}^+\}$ can be accepted by a Turing machine .

F Any context free grammar can be converted to an NPDA that accepts the same language.

T The grammar $S \rightarrow aSbbb|b$ generates the language $L = \{a^n b^{3n+1} : n \geq 0\}$.

T The language $L(a^*b^*aa(a+b)^*)$ can be accepted by a Turing machine.

Section 2. (5 points each)

3.5

1. Consider the following language

$$L = \{a^n b^\ell : 3n \leq \ell \leq 5n, n \geq 0\}.$$

Prove that this language is context-free.

$-1$

$n = 1$ $3 \leq \ell \leq 5$ 3, 4, 5

$S \rightarrow aSbbb \mid aSbbbbb \mid B$ $aSb^5$ ?
$B \rightarrow b \mid \lambda \mid \cancel{aBb}$

X $-\frac{1}{2}$

abbb abbbbb abbbbb

$n=2$ $6 \leq \ell \leq 10$ aabbbbbb

aSbbb
aaSbbbbbb
aa$\lambda$bbbbbb
aabbbbbb

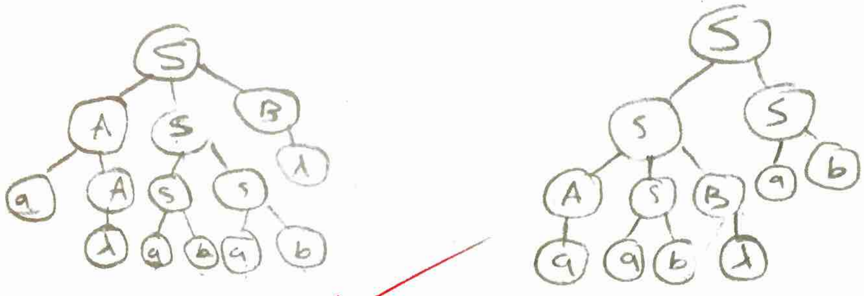
∴ There is context free grammer
∴ The language is context-free

2. Show that the following grammar is ambiguous.

$$S \to ASB|ab|SS$$
$$A \to aA|\lambda$$
$$B \to bB|\lambda$$

Let w = aabab

5




w = aabab

w = aabab

∵ 2 derivation trees generate the same word

∴ it is ambiguous

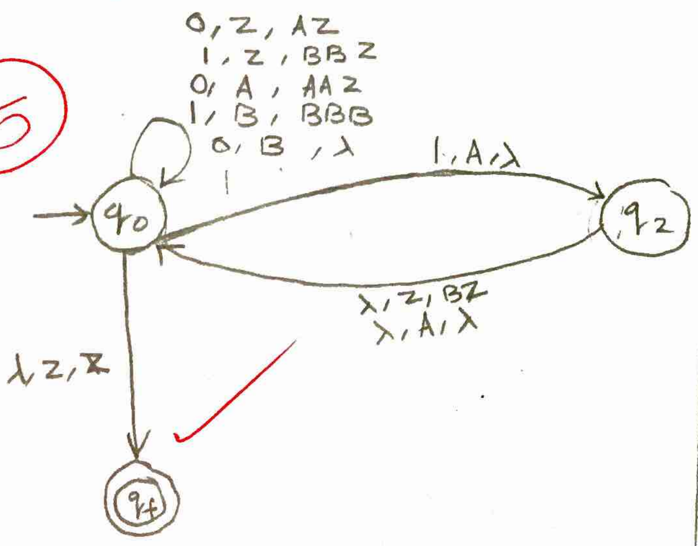
3. A signal is received in a remote weather station. The signal contains a sequence of $n$ bits. In order to check if the signal contains errors the sending station encodes it so that the number of 0's is always twice the number of 1's.

Construct an NPDA that will check weather a given sequence of bits is acceptable or not according to this error checking scheme. Show your work and list your assumptions, if any.

0 a
1 b

no. of 0's = 2 no 1's
no of a's = 2 no b's

5




no. 1's = 3
no. 0's = 6

010011000


no. of 1 = 2
no of 0 = 4

010100

4. Convert the following context-free grammar to an equivalent NPDA that accepts the same language.

$$\begin{aligned} S &\rightarrow aaAB|bSb \\ A &\rightarrow bA|C \\ B &\rightarrow CBa|a \\ C &\rightarrow ab|b \end{aligned}$$

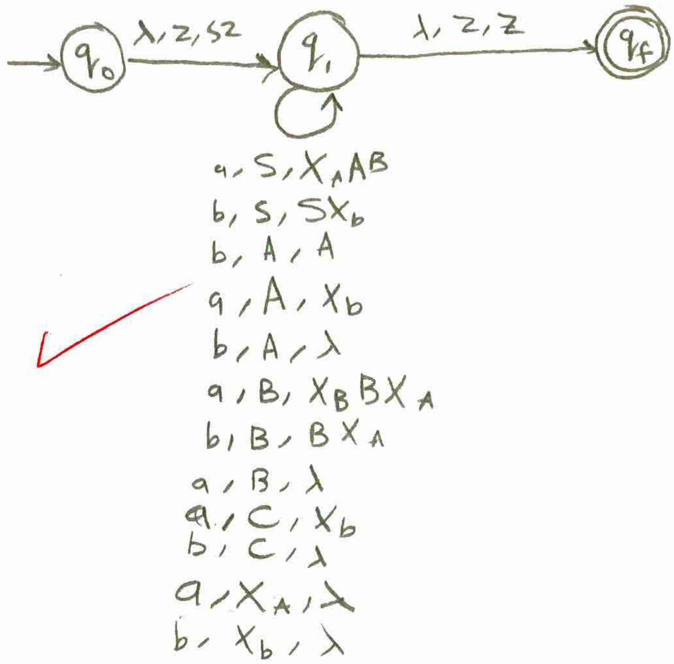
5

① Convert to GNF

$S \rightarrow aabAB \mid aaCB \mid bSb$
$A \rightarrow bA \mid C$
$B \rightarrow abBa \mid bBa \mid a$
$C \rightarrow ab \mid b$

$S \rightarrow aX_AAB \mid bSX_B$
$A \rightarrow bA \mid ab \mid b$
$B \rightarrow abBa \mid bBa \mid a$
$C \rightarrow ab \mid b$

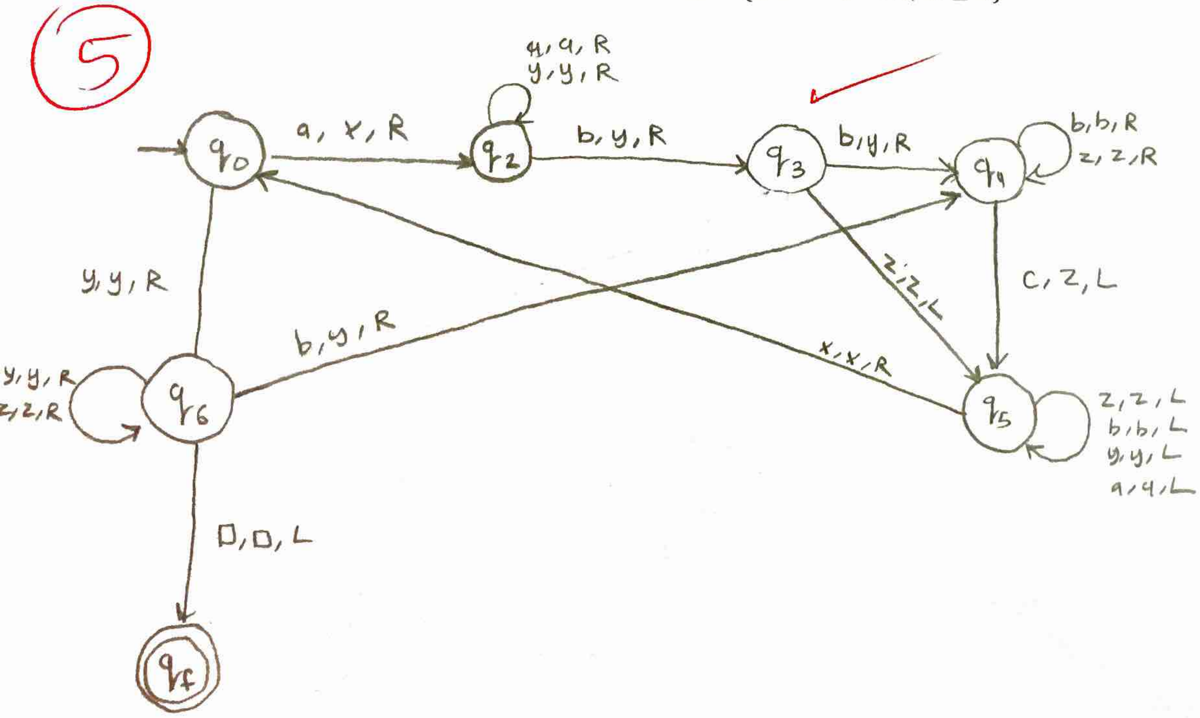
$$\boxed{\begin{aligned} &S \rightarrow aX_AAB \mid bSX_B \\ &A \rightarrow bA \mid aX_b \mid b \\ &B \rightarrow aX_BBX_A \mid bBX_A \mid a \\ &C \rightarrow aX_b \mid b \\ &X_A \rightarrow a \\ &X_b \rightarrow b \end{aligned}}$$




5. Construct a Turing machine that accepts the following language.

$$L = \{a^n b^{n+m} c^m : n, m \geq 0\}$$

5




aa bbbbbbb ccccc
xx yyyyyyy zzzzz

aaa bbbbb cc
xxx yyyyy zz

aaaa bbbbbbbccc
xxxx yyyyyyyzzz

aa bbbb cc
xx yyyy zz

aabbbc
xx yyy z